Tensegrity Engineering
for
Space Systems

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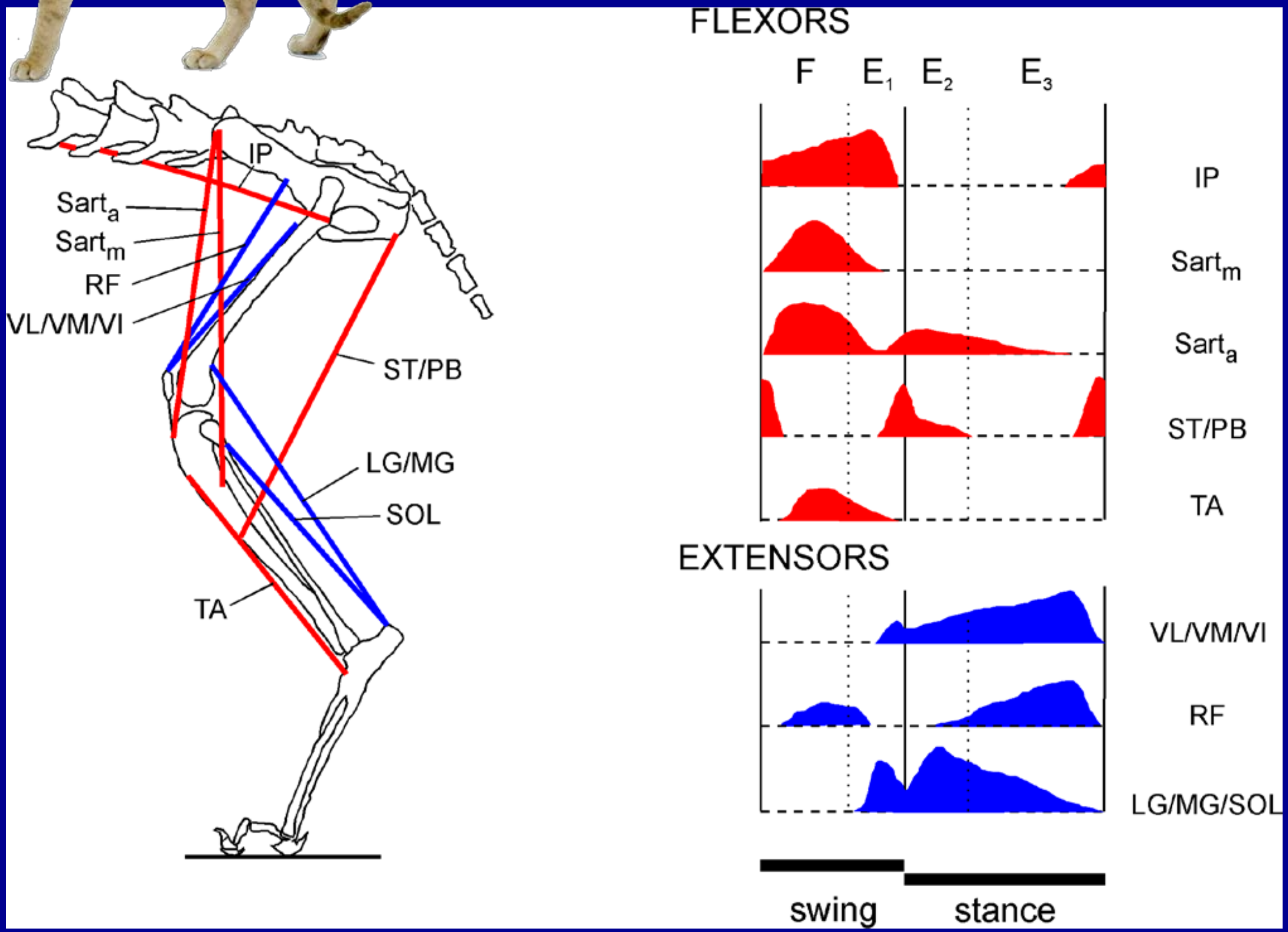
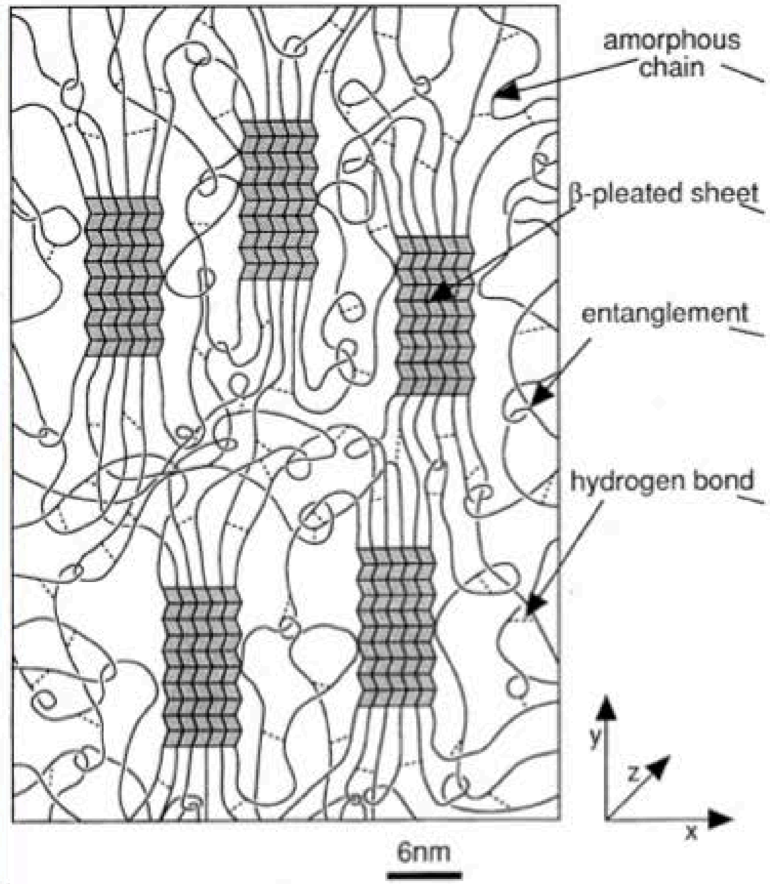
First Motivation: Animal Locomotion

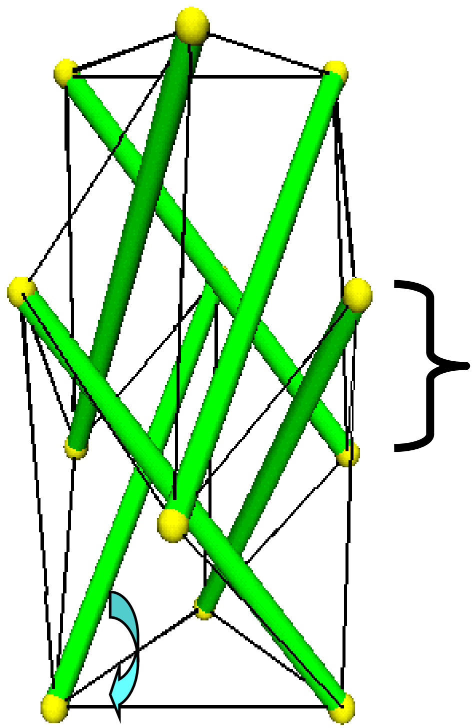


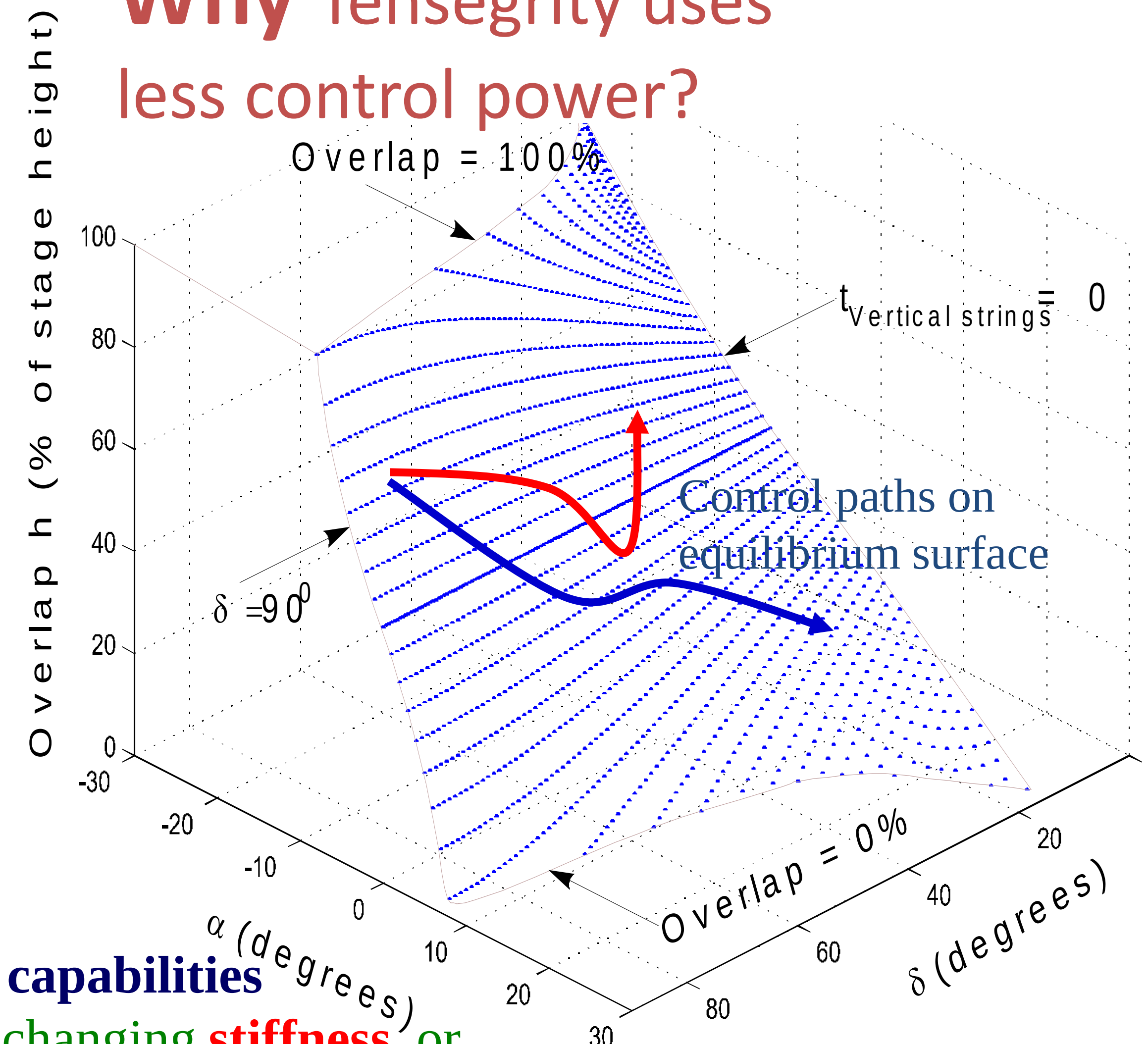
The dragline silk of a Nephila Clavipes



The molecular structure of nature's strongest fiber.
The rigid bodies are the β-pleated sheets, and the tensile members are the amorphous strands that connect to the β-pleated sheets

Why Tensegrity uses less control power?





Two unique tensegrity capabilities

-Change **shape** without changing **stiffness**, or

-Change **stiffness** without changing **shape**

Class 1 Tensegrity Dynamics

Let rigid rods and massless elastic strings be connected as

$$\begin{bmatrix} B & R & S \end{bmatrix} = N \begin{bmatrix} C_b^T & C_r^T & C_s^T \end{bmatrix}, \qquad Then$$

$$\ddot{N}M + NK(\gamma, \dot{N}, W) = W$$

$$M = C_b^T \hat{m} C_b \frac{1}{12} + C_r^T \hat{m} C_r, \qquad K(\gamma, \dot{N}, W) = C_s^T \hat{\gamma} C_s - C_b^T \hat{\lambda} C_b$$

where

$$-\hat{\lambda} = \lfloor \dot{B}^T \dot{B} \rfloor \hat{m} \hat{l}^{-2} \frac{1}{12} + \lfloor B^T F(\gamma) C_b^T \rfloor \hat{l}^{-2} \frac{1}{2}, \qquad F(\gamma) = W - S\hat{\gamma}C_s$$

$$\lambda_i = \frac{\text{force in bar } b_i}{\|b_i\|}, \qquad \gamma_i = \frac{\text{force in string } s_i}{\|s_i\|}$$

Active Form-Finding

Feedback Control to converge to a specified desired shape $\bar{Y}$:

$$\ddot{N}M + NK(\gamma) = W\ , \qquad K(\gamma) = C^T \Sigma C, \qquad Y(t) = LN(t)R = \text{current shape}$$

$$\Sigma = \begin{bmatrix} -\hat{\lambda} & 0 \\ 0 & \hat{\gamma} \end{bmatrix}, \qquad \begin{bmatrix} B & S \end{bmatrix} = NC^T, \quad C^T = \begin{bmatrix} C_b^T & C_s^T \end{bmatrix},$$

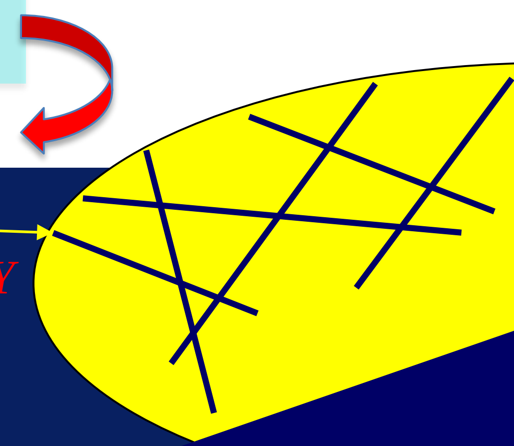
$$\hat{\lambda} = \frac{1}{12}\hat{l}^{-2}\left\lfloor 6B^T(W - S\hat{\gamma}C_s)C_b^T - \dot{B}^T\dot{B}^T\hat{m} \right\rfloor \ = \text{a diagonal matrix}$$

Control objective: Y(t) $\rightarrow \bar{Y}$.

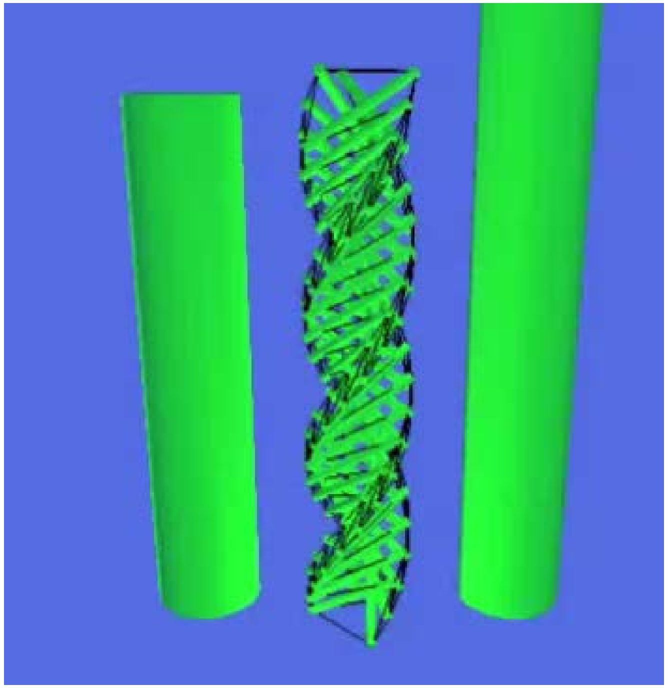
Find the control γ to cause the error $\Omega(t)$=Y(t)-$\bar{Y}$

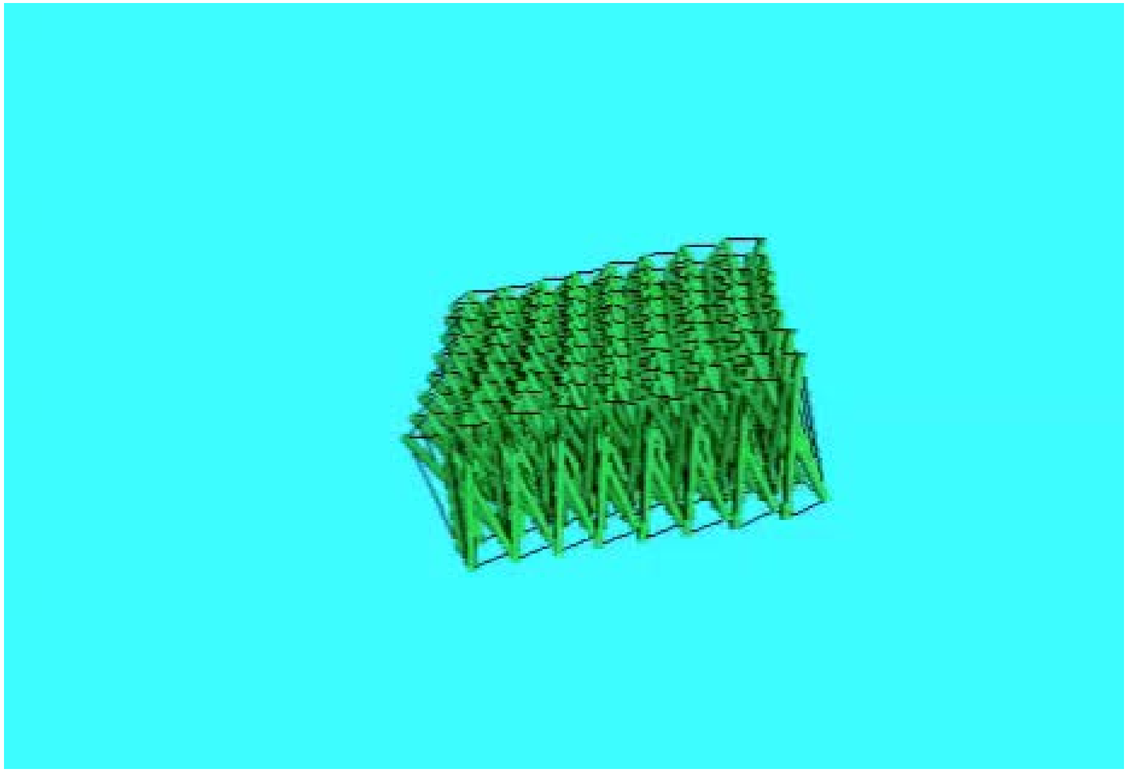
to satisfy a stable eq $\ddot{\Omega}+P\dot{\Omega}+Q\Omega=0$

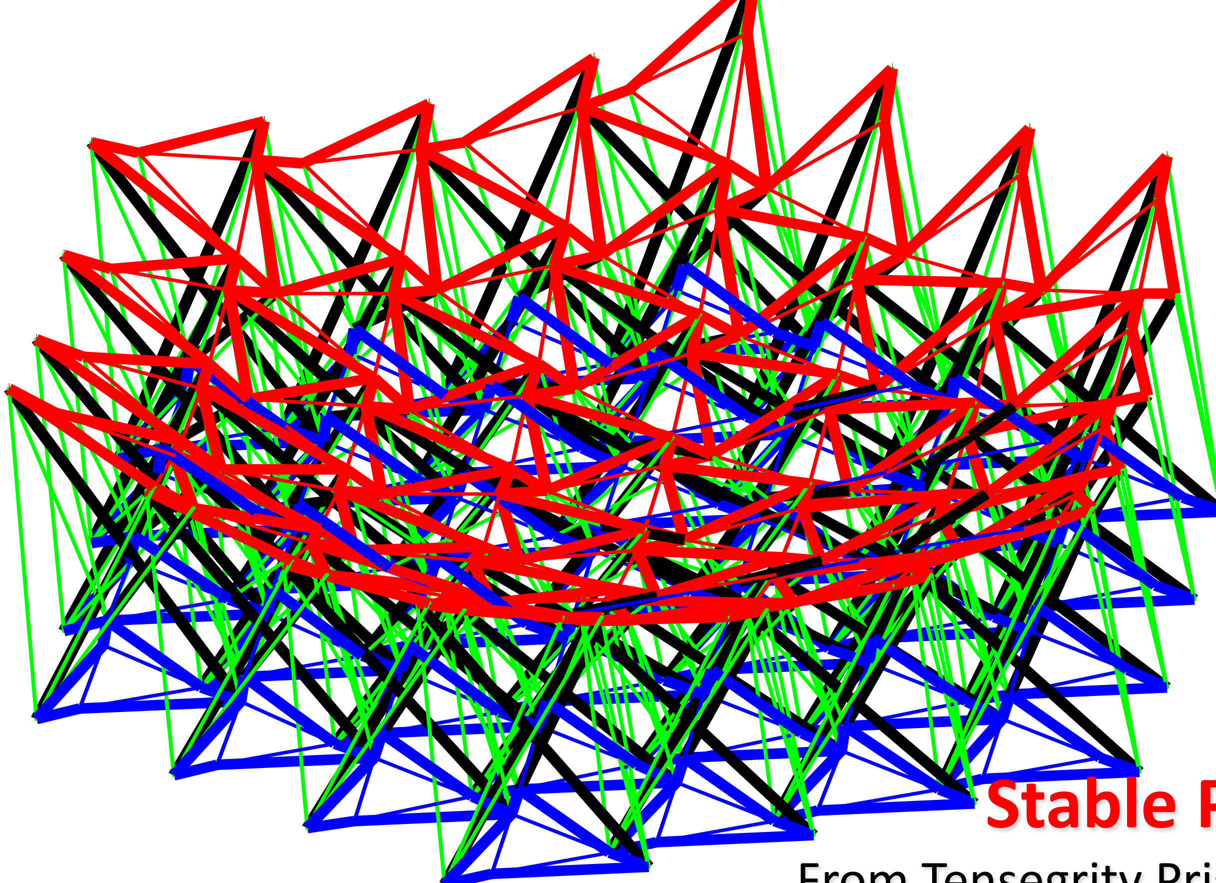
LINEAR in $\gamma(N(t), \dot{N}(t), W(t))$!!



Desired shape
(Location of selected nodes) Y







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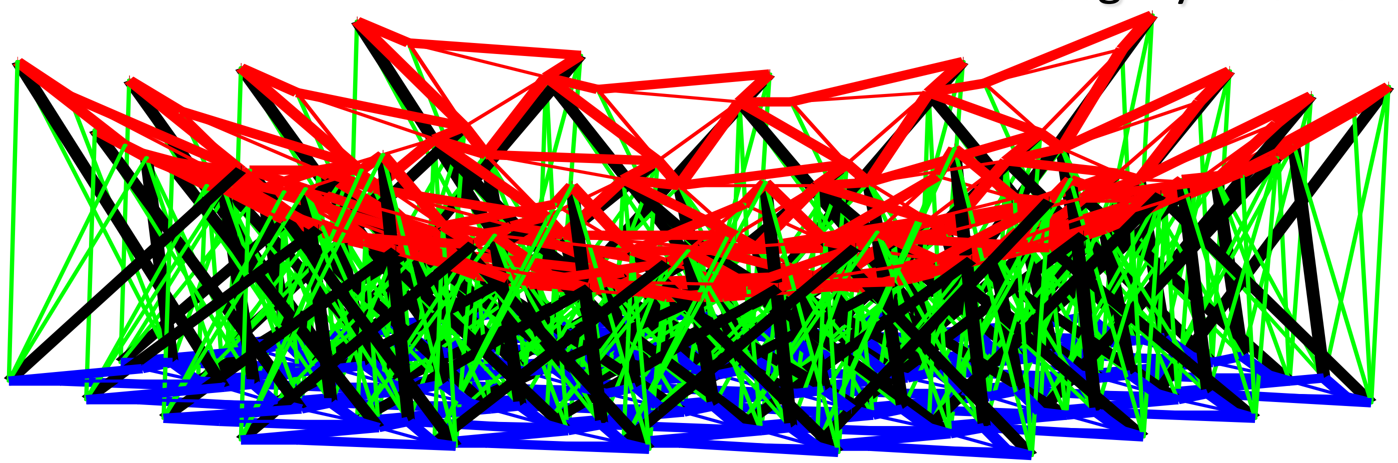
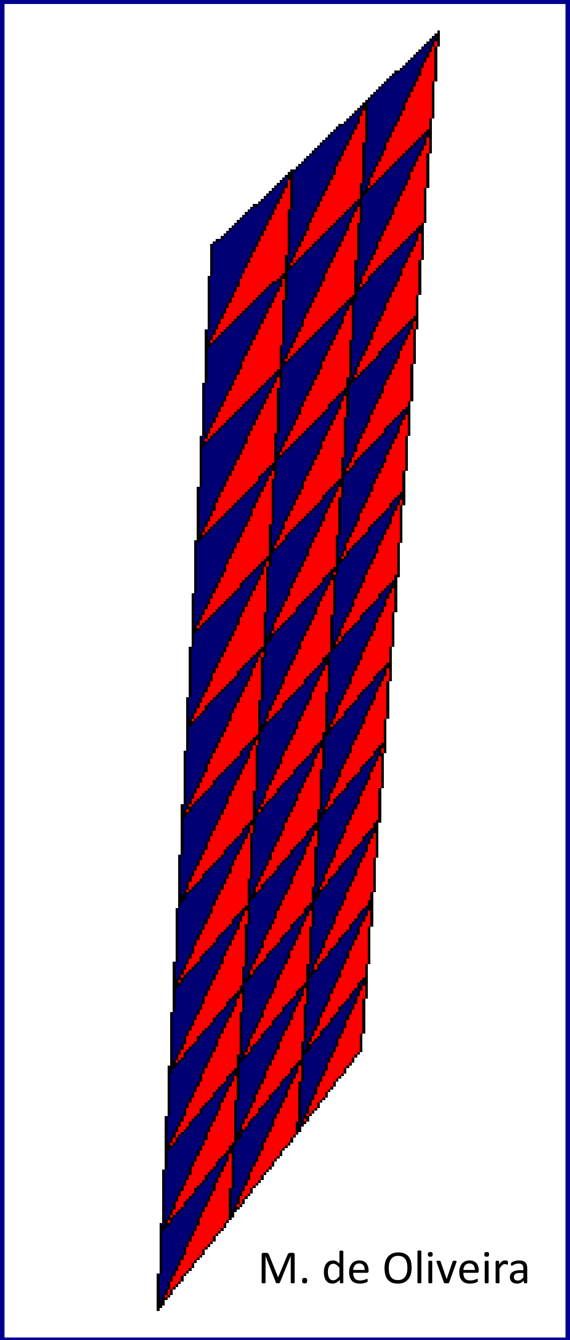
Stable Parabolic Surface

From Tensegrity Prisms with no soft modes





Integrate Design of Origami/Tensegrity

Tensegrity: 3D structures from 1D objects

(High strength, high stability, low mass, deployable)

Origami: 3D structures from 2D objects

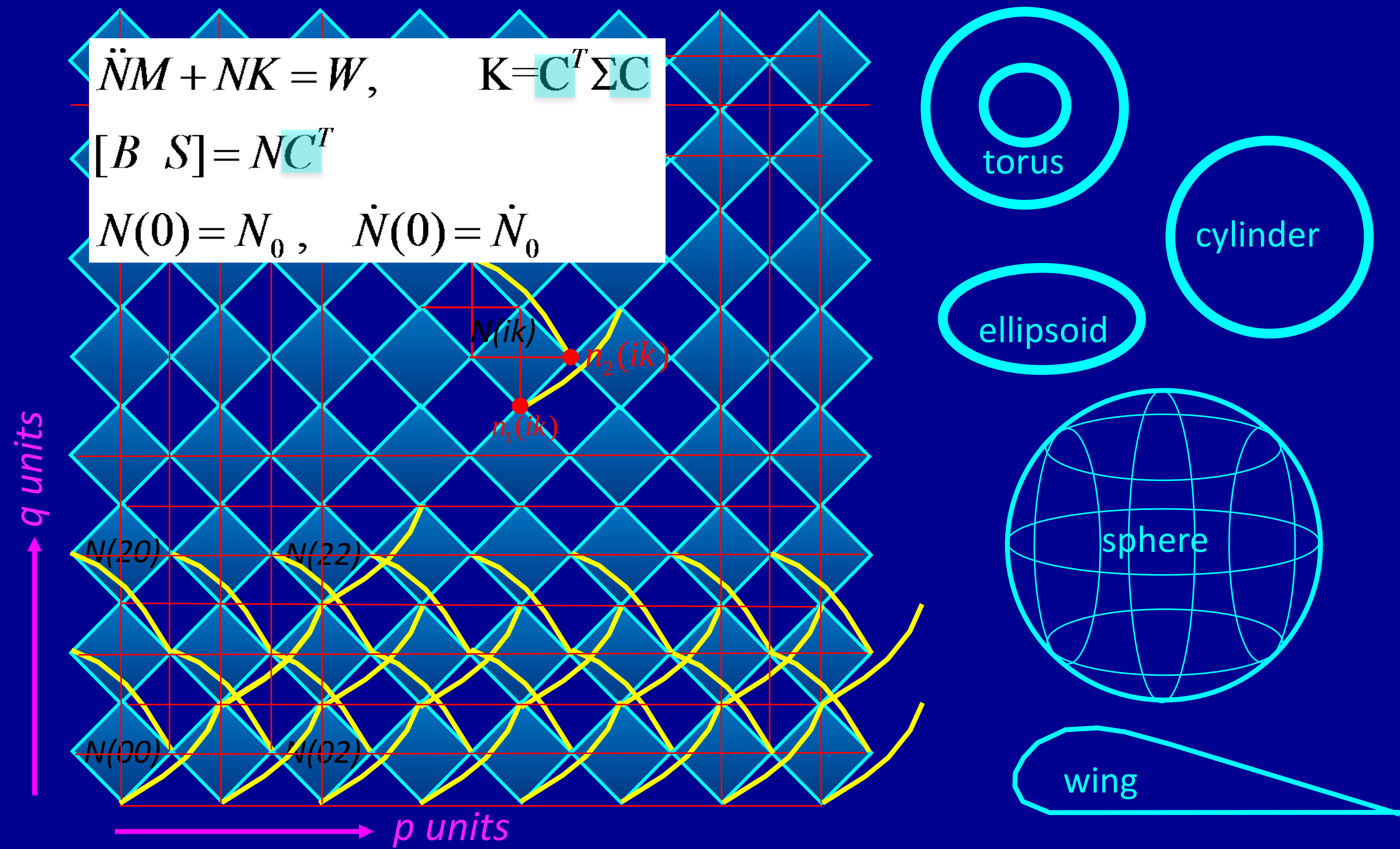
(low strength, low stability, foldable)

Double Helix Tensegrity (DHT)

$$\ddot{N}M + NK = W, \qquad K = C^T \Sigma C$$

$$[B \;\; S] = NC^T$$

$$N(0) = N_0\,, \quad \dot{N}(0) = \dot{N}_0$$



Torus

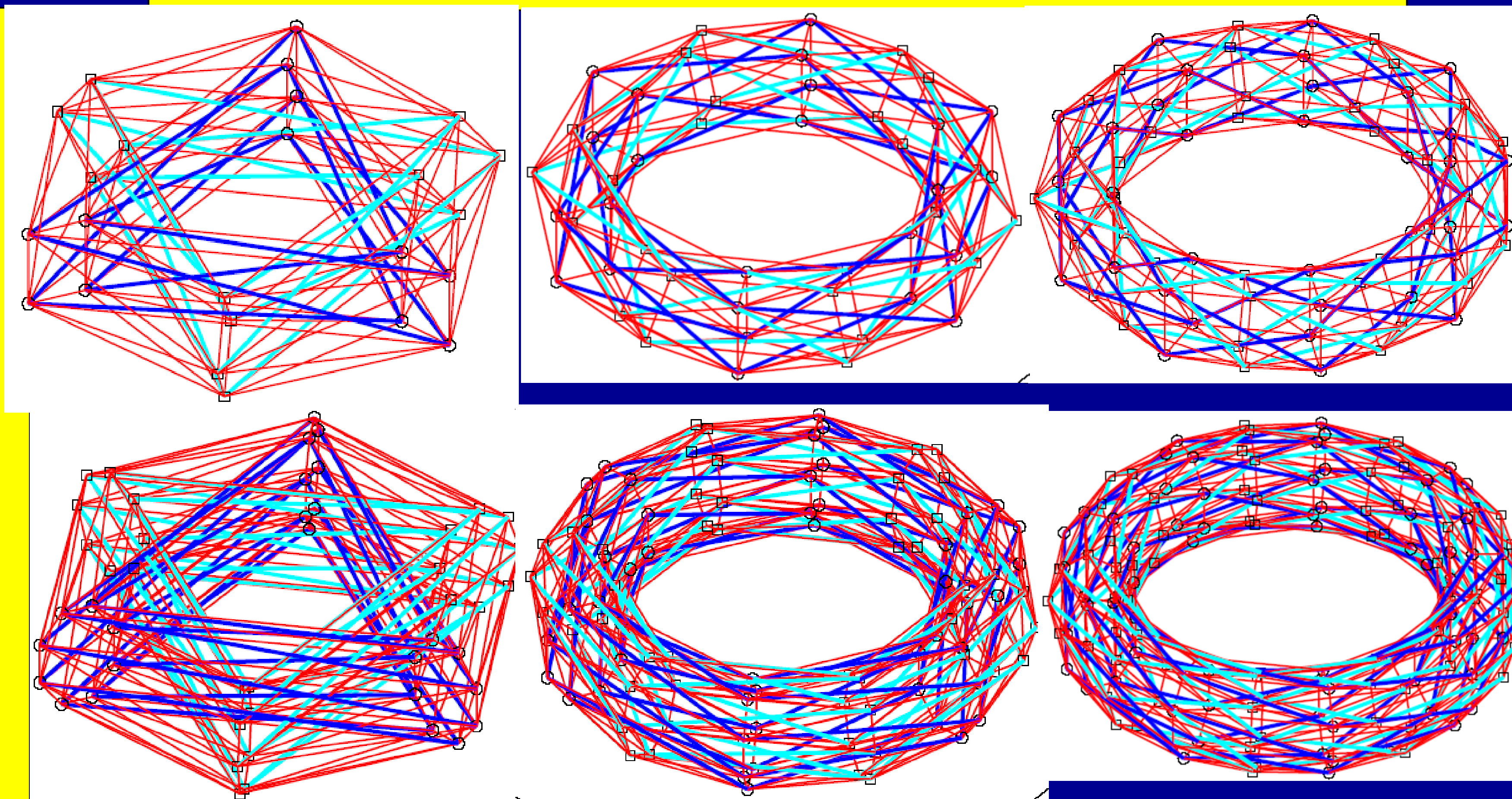
i → , k ↓

$R = 1, r = 0.2$

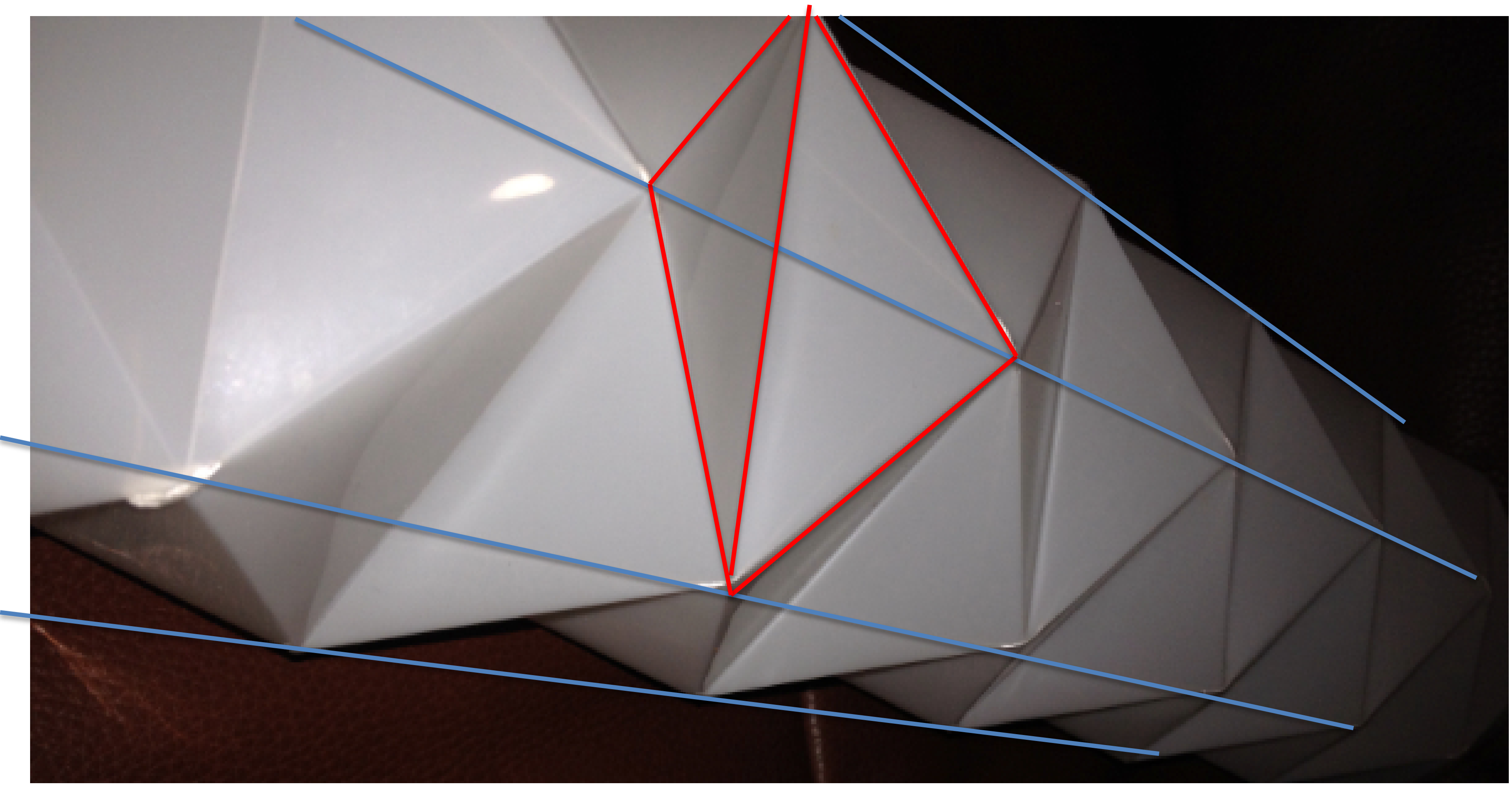
$q = 3$ | $q = 6$ | $q = 9$

$p = 4$

$p = 8$



Double Helix Tensegrity (DHT): Exterior View



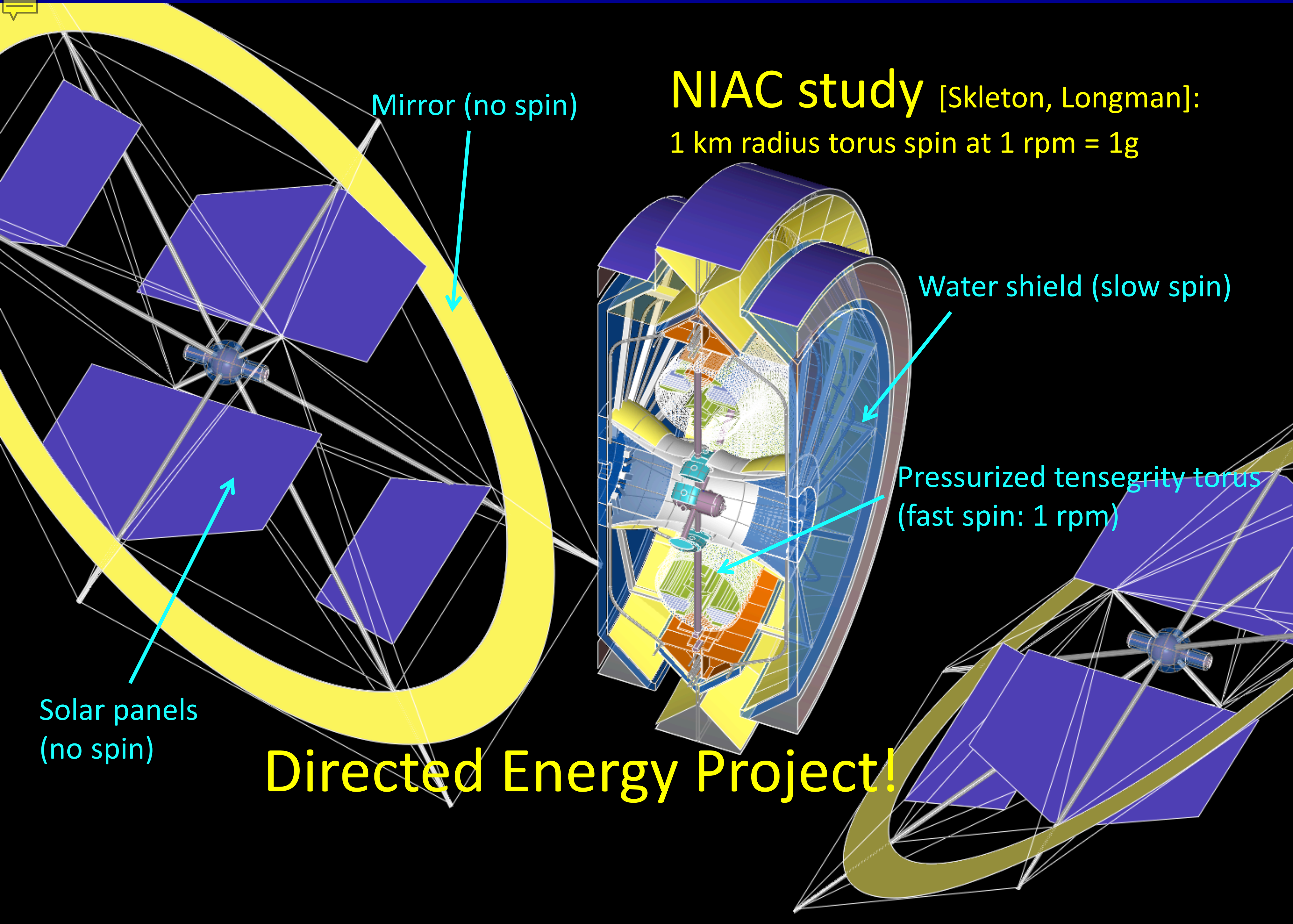
Primal and Dual DHT

Primal DHT: White lines are compressive members
Edges are tensile members

Dual DHT: White lines are tensile members (cables)
Edges are compressive members

NIAC study [Skleton, Longman]:

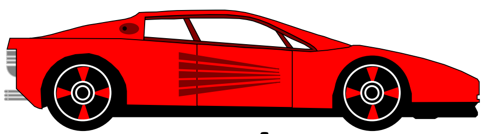
1 km radius torus spin at 1 rpm = 1g



Directed Energy Project!

We Need Tools to Pin the Tail on the Performance Limiting Technology

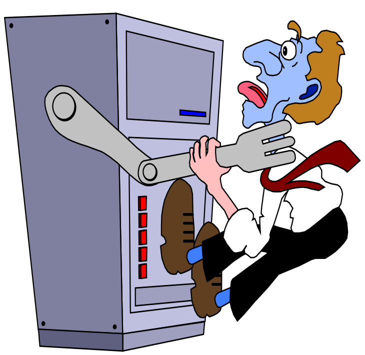
From first principles, Universities teach component technology

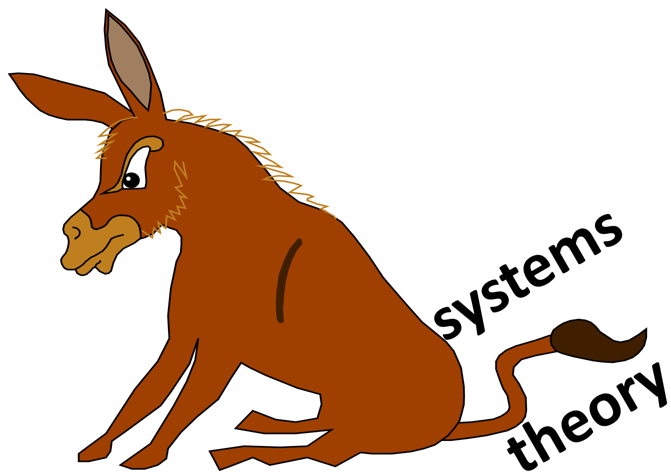


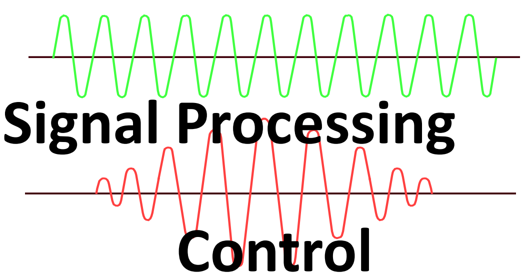
Design/Mfg

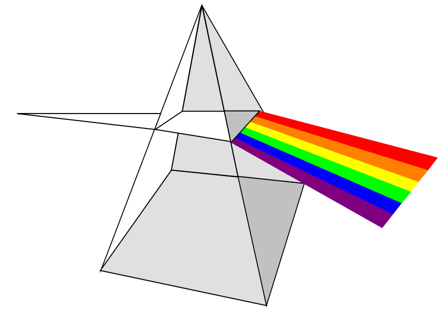
Modeling

Computing









Physics

Sensing

Critical Issues in System Design

Given a performance bound we seek to:

- Min Mass of structure
- Min Energy for control

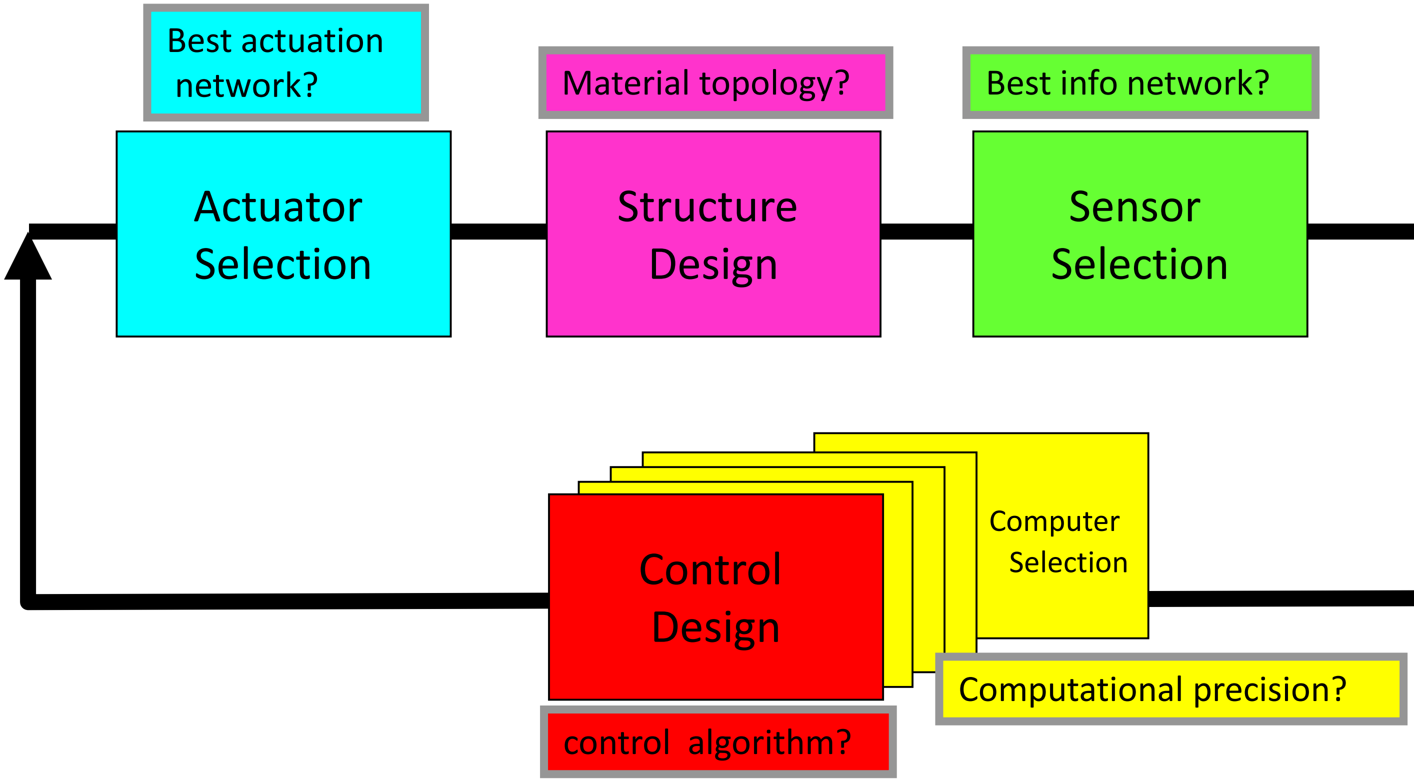
} MME design

  - Enabling technologies for MME design:
    - Minimal mass structures (tensegrity)
    - Information Architecture (integrate choice of sensor/actuator networks, sensor & computational precisions, and control or estimation laws)
    - Deployment schemes (origami/tensegrity)
    - Model improvements from data

Information Architecture

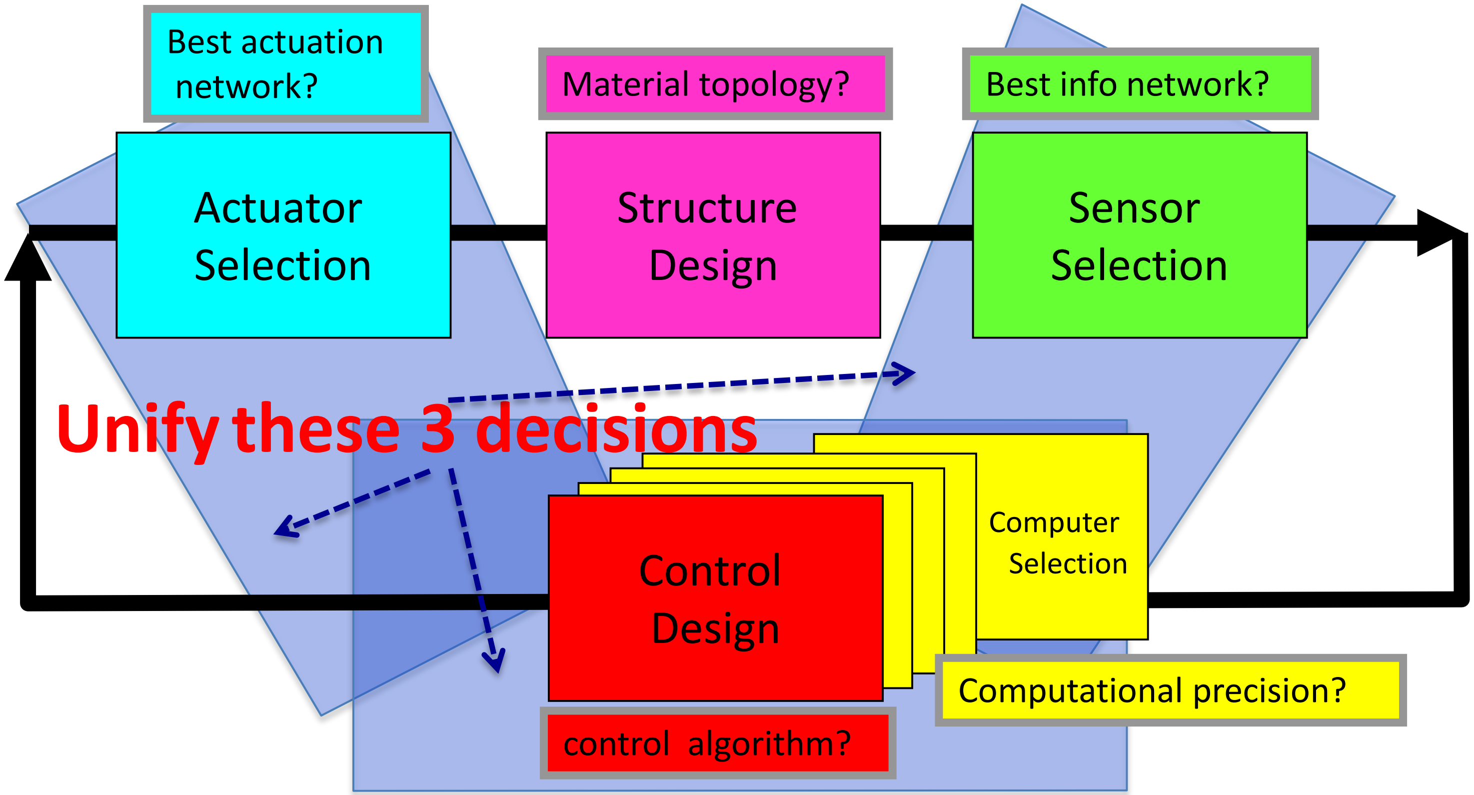
What to measure/actuate/compute?
With what precision?
With what control law?
With what cost?

How to integrate these choices?



Information Architecture

(What to measure/actuate/compute? With what precision?
With what control law? With what cost?)



Information Architecture [Skelton 2009] jointly optimizes:

1) control law and 2) sensor/actuator network

(proves this is a convex problem if dynamics linear)

Conclusions
(the set of new analytical tools)

- Control theory has been extended to: select the precision and location required of all instruments while satisfying prespecified bounds on:
  - Total instrument costs
  - Performance errors
  - Control energy
- Analytical tools are available to: integrate origami and tensegrity designs of deployable structures, with shape control.
- Analytical tools are available to: generate all linear models which can identically match the data (a specified number of autocorrelations and cross-correlations of input/output data)

Information Architecture and Control

Plant: $\dot{x}_p = A_p x_p + B_p u + D_p w$

Output: $y = C_p x_p + B_y u$

Measurement: $z = M_p x \qquad + D_z w$

$$E\begin{bmatrix} w_a \\ w_s \\ w_p \end{bmatrix} = 0,\quad E\begin{bmatrix} w_a(t) \\ w_s(t) \\ w_p(t) \end{bmatrix}\begin{bmatrix} w_a(\tau) \\ w_s(\tau) \\ w_p(\tau) \end{bmatrix}^T = \begin{bmatrix} W_a & 0 & 0 \\ 0 & W_s & 0 \\ 0 & 0 & W_p \end{bmatrix}\delta(t-\tau)$$

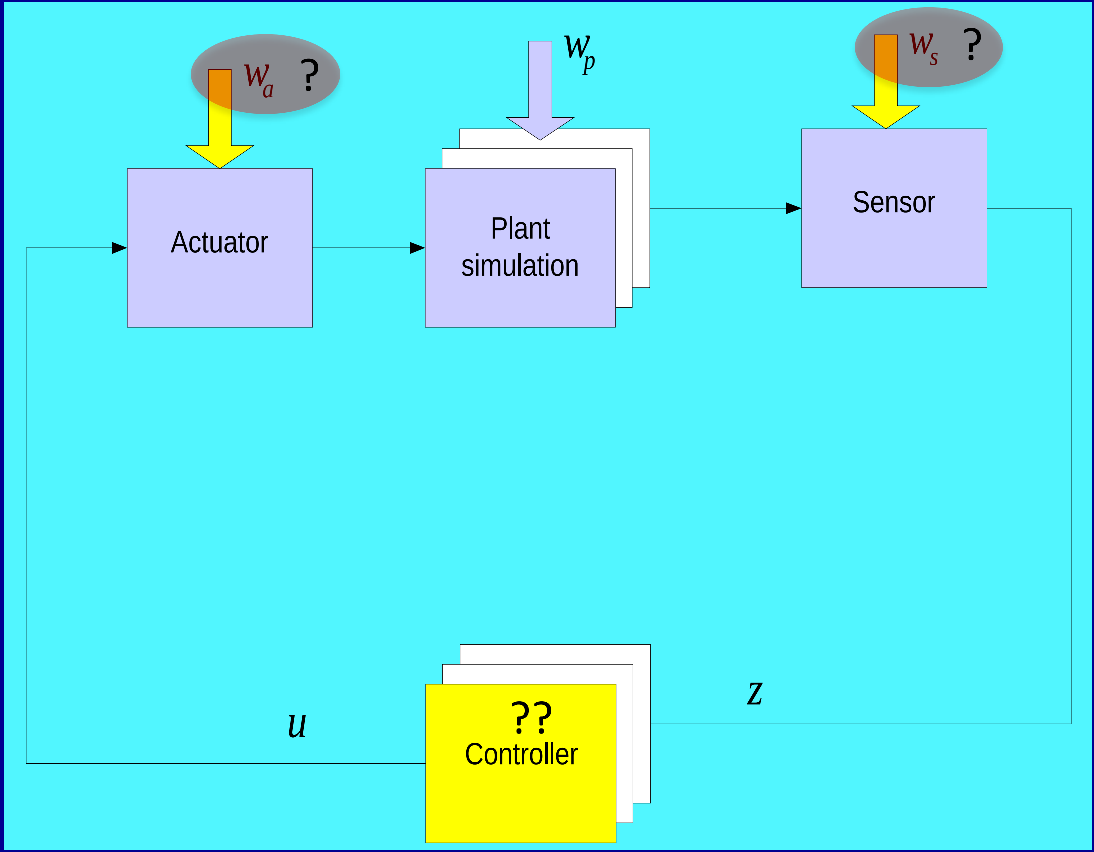
Controller G: $\dot{x}_c = A_c x_c + B_c z$, $u = C_c x_c + D_c z$

Find G such that

$$E\left[uu^T\right] < \bar{U}$$

$$E\left[yy^T\right] < \bar{Y}$$

} convex



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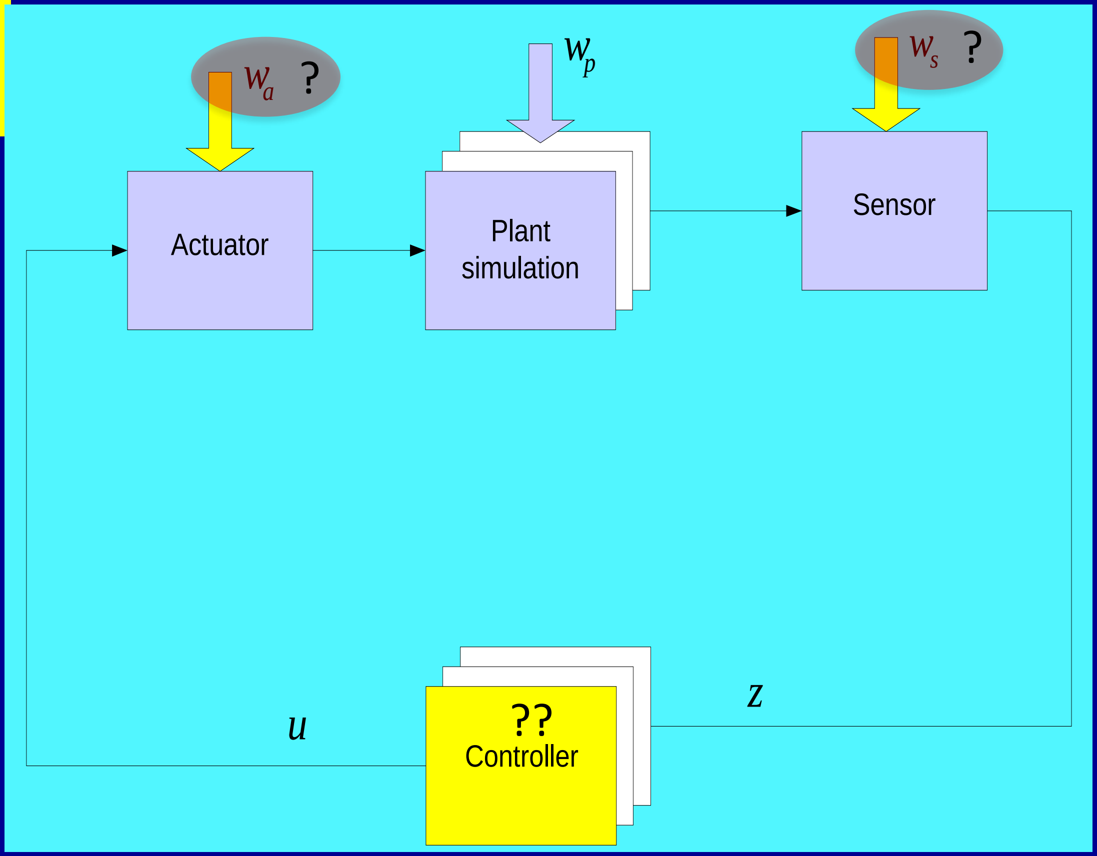
$$\$:= trPW^{-1} \qquad W^{-1} := \begin{bmatrix} W_a^{-1} & 0 \\ 0 & W_s^{-1} \end{bmatrix}$$

Controller **G**: $\dot{x}_c = A_c x_c + B_c z$, $u = C_c x_c + D_c z$

Find W and G such that

$$\left.\begin{aligned} &\$ < \overline{\$} \\ &E\left[uu^T\right] < \overline{U} \\ &E\left[yy^T\right] < \overline{Y} \end{aligned}\right\} \text{ Convex!}$$



Sensor/Actuator Network Selection

$\$ \leq \bar{\$}$ Money

$E[yy^T] \leq \bar{Y}$ Performance

$E[uu^T] \leq \bar{U}$ Energy

Convex problem

$$W^{-1} = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \ddots \end{bmatrix}$$

Instrument precisions

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_n$$

After solving convex problem
Observe this ranking

Delete smallest precision instrument and repeat convex problem with smaller set of Instruments .
Stop when feasibility is lost